High-Efficient Ranging Algorithms for Wireless Sensor Network

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***Abstract*—Obtaining precise internode range information with high-efficiency is essential for the Internet of Things. Two-way ranging (TWR) algorithm and its variants are widely used for measuring time-of-flight (ToF) among wireless transceivers. However, these algorithms are inefficient for large-scale networks, due to high resource consumption and long latency. In this paper, for active nodes and passive nodes, we propose the network-based TWR (NB-TWR) and the network-based passive ranging (NB-PR) algorithms, respectively, which realize all internode ToFs measuring with the minimum frames. Through theoretical analysis, we show that the ranging errors of the proposed methods are much lower than those of traditional algorithms. Our results can serve as guidelines for wireless networks to realize network ranging with high precision and limited resource consumption.**

***Index Terms*—two-way ranging, network ranging, passive ranging, time-of-flight, localization**

I. Introduction

The Internet of Things (IoT) evolves from the convergence of wireless technology and micro-electromechanical system. Recently, providing precise positioning service for wireless sensor networks (WSNs) occupies increasing attentions [1]–[3]. Traditional two-way ranging (TWR) algorithm and its variations, e.g., single-sided TWR (SS-TWR), double TWR (D-TWR) [4], double-sided TWR (DS-TWR) [5], asymmetric double-sided TWR (ADS-TWR) [6] and alternative double-sided TWR (AltDS-TWR) [7], are used to measure the time-of-flight (ToF) between agents and anchors, in which each agent has at least three times frame exchange with the anchor.

Nevertheless, for an N-node network, traditional two-way algorithms need a large amount of frames to acquire ranging information between each pair of nodes, which are on the order of magnitude at $\mathcal{O}(N^2)$. In order to save time and radio resources, we adopt the idea of frame multiplexing to reduce frame costs. Depending on whether the node sends frames, we name the nodes that participate in frame exchanges as active nodes and those only listening to other nodes' frames without message transmitting as passive nodes. Furthermore, we denote the network with N active nodes as N-active network.

For an N-active network, we propose the network-based TWR (NB-TWR) algorithm, which includes redesigning the communication protocol and ToF estimation. With the NB-TWR, determining all $\binom{N}{2}$ ToFs among the active nodes with high accuracy only requires $N+1$ frames. This algorithm will be introduced in Section III in detail. Moreover, for WSNs with passive nodes, we propose the network-based passive ranging (NB-PR) algorithm, which enables an unlimited number of passive nodes to realize self-localization. Compared with existing passive ranging methods, such as passive extended ranging (PER) [8] and sequential time difference of arrival (S-TDOA) [9], the proposed NB-PR can additionally achieve clock synchronization between the active nodes and the passive nodes. The NB-PR algorithm will be introduced in Section IV.

By combining the NB-TWR and the NB-PR, we propose a high-efficient ranging strategy for WSNs, of which the estimation errors are in the same order of magnitude as the current highest accuracy, i.e., the accuracy of the AltDS-TWR [10]. The theoretical analysis and numerical simulation are provided in Section V and VI, respectively.

Notation: the notation V and $\hat{V}$ denote the value in real and typical cases, respectively. We use both English letters and arabic numerals to number nodes, and further define the function $\mathbb{N}(\alpha)$ and $\mathbb{N}^{-1}(i)$ for transformation. (e.g. $\mathbb{N}(A)=1$ and $\mathbb{N}^{-1}(1)=A$). Meanwhile, for convenience and readability, we define $\sum\limits_{k=2}^{\mathbb{N}(\beta)}$ and $\sum\limits_{k=\mathbb{N}(\alpha)+1}^{\mathbb{N}(\beta)}$ to denote $\sum\limits_{k\in\mathcal{L}_\beta}$ and $\sum\limits_{k\in\{\mathcal{L}_\beta/\mathcal{L}_\alpha\}}$, respectively, where $\mathcal{L}_\beta$ is the integer set from 2 to $\mathbb{N}(\beta)$ and $\{\mathcal{L}_\beta/\mathcal{L}_\alpha\}$ is the integer set from $\mathbb{N}(\alpha)+1$ to $\mathbb{N}(\beta)$.

II. System Model

Consider a wireless sensor network (WSN) where each node equips with an unsynchronised clock. Assume the clocks of different nodes are independent. To realize network ranging, a node mark down a timestamp once transmits or receives a message. Let X denote the node in the network and $t_{X,i}$ denote the ith timestamp captured by node X. Due to clock asynchronism, the timestamps cannot be accurately captured [11] and according to the standard IEEE 802.15.4, the imperfection in timing references is mainly due to the clock drift [12]. Hence, in an independent ranging process, the relationship between the true timestamp $t_{X,i}$ and the measured timestamp $\hat{t}_{X,i}$ is modelled as

$$\hat{t}_{X,i}=k_X t_{X,i}=(1+e_X)t_{X,i}, \tag{1}$$

where e_X denotes the deviation from real frequency and is typically expressed in parts per million (ppm). With two timestamps, the time delay can be described as

$$\hat{\tau}_{X,i}=\hat{t}_{X,i+1}-\hat{t}_{X,i}=k_X(t_{X,i+1}-t_{X,i})=k_X\tau_{X,i}, \tag{2}$$

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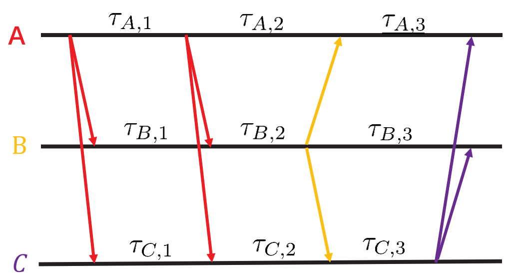

Fig. 1: An illustration of the proposed communication protocol for ranging among three active nodes.

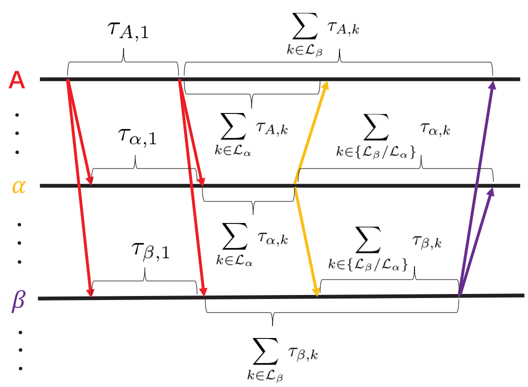

Fig. 2: Using the proposed NB-TWR algorithm to determine ranges among multiple active nodes.

where $\hat{\tau}_{X,i}$ and $\tau_{X,i}$ are the true and the ideally measured time delay, respectively.

In the next two Sections, we first introduce a novel communication protocol for ranging, and then propose two internode ranging algorithms, NB-TWR and NB-PR, for active nodes and passive networks, respectively.

III. Network Based Two-Way Ranging Algorithm

We reduce the time and frame costs of network ranging by reusing frames. Note that when a node gathers all timestamps information, then with one more broadcasting frame, every node in the network will know such information as well. Inspired by this, we propose a novel communication protocol for ranging, where the illustrations of the protocol for three nodes and multiple nodes are shown in Fig. 1 and Fig. 2, respectively. The pseudocode of the corresponding ranging algorithm is shown in Algorithm 1 and the execution process is described as follows.

Algorithm 1 Network based two-way ranging

Input: The number of active nodes N, $N+1$ frames, the id set of active nodes $\mathcal{C}$, preset synchronization time $\tau_{A,1}$ response delay $\tau_{X,\mathbb{N}(X)}$ where $X \in \mathcal{C}$ & $X \neq A$.

Output: Estimated ToF $\hat{T}_{of}(\alpha,\beta)$ where $\alpha,\beta \in \mathcal{C}$.

```
The node A transmits the fisrt message;
After the synchronization time τ_{A,1}, A transmits the second message;
i ← 2 ;
while i ≠ N + 1 do
    X ← ℕ^{-1}(i) ;
    Delay τ_{X,i};
    X transmits a message;
    i ← i + 1;
end while
A calculates all T̂_{of}(α, β) where α, β ∈ C by fomula (3);
return All T̂_{of}(α, β);
```

Firstly, a ranging-request message is broadcasted by an arbitrary active node. Without loss of generality, we call it as node A. Then, after a given delay $\tau_{A,1}$, another message is broadcasted by A to initiate a reply request. We name $\tau_{A,1}$ as synchronization time. Then, the other nodes respond the node A in turn. The reply frames contain all timestamps that the transmitter possesses at that time. Note that every message can be received by all nodes. The algorithm stops until all nodes respond already. Next, the node A can calculate all ToFs by (3). Let $T_{of}(\alpha,\beta)$ and $\hat{T}_{of}(\alpha,\beta)$ denote the true ToF and the estimated ToF between the node α and β, respectively.

Proposition 1: The ToF of the wireless network with NB-TWR algorithm can be calculated by (3).

Proof: To better explain (3), we take the simplest 3-node case as an example. As shown in Fig 1, (1) indicates that

$$\frac{\hat{\tau}_{A,1}}{\hat{\tau}_{X,1}} = \frac{k_A \tau_{A,1}}{k_X \tau_{X,1}} = \frac{k_A}{k_X}, \tag{4}$$

where X denotes the id of the other nodes, i.e., $X \in \{B, C\}$. Firstly, consider the ToF $T_{of}(A,B)$. Since $T_{of}(A,B) = \frac{1}{2}(\tau_{A,2} - \tau_{B,2})$, we can estimate $T_{of}(A,B)$ by

$$\hat{T}_{of}(A,B) = \frac{1}{2}(\hat{\tau}_{A,2} - \hat{\tau}_{B,2}). \tag{5}$$

However, the estimation result of (5) has an unaccepted error [7], due to the frequency deviation entailed by clock drift. To

$$\hat{T}_{of}(\alpha,\beta) = \begin{cases} \dfrac{\sum\limits_{k\in\mathcal{L}_\beta} (\hat{\tau}_{\beta,1}\hat{\tau}_{A,k} - \hat{\tau}_{A,1}\hat{\tau}_{\beta,k})}{\hat{\tau}_{A,1} + \hat{\tau}_{\beta,1}}, & \mathbb{N}(\beta) \geq \mathbb{N}(\alpha) = 1 \\ \dfrac{3\left[\hat{\tau}_{A,1} \sum\limits_{k\in\mathcal{L}_\alpha} (\hat{\tau}_{\alpha,1}\hat{\tau}_{\beta,k} - \hat{\tau}_{\beta,1}\hat{\tau}_{\alpha,k}) + \hat{\tau}_{\alpha,1} \sum\limits_{k\in\{\mathcal{L}_\beta/\mathcal{L}_\alpha\}} (\hat{\tau}_{\beta,1}\hat{\tau}_{A,k} - \hat{\tau}_{A,1}\hat{\tau}_{\beta,k})\right]}{2(\hat{\tau}_{A,1}\hat{\tau}_{\alpha,1} + \hat{\tau}_{\alpha,1}\hat{\tau}_{\beta,1} + \hat{\tau}_{A,1}\hat{\tau}_{\beta,1})}, & \mathbb{N}(\beta) \geq \mathbb{N}(\alpha) > 1 \\ \hat{T}_{of}(\beta,\alpha), & \mathbb{N}(\beta) < \mathbb{N}(\alpha) \end{cases} \tag{3}$$

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mitigate the deviation, we measure all delay estimations by utilizing the reference node's clock drift as

$$\hat{\tau}'_{B,2} \triangleq k_A \tau_{B,2} = \frac{k_A}{k_B}\hat{\tau}_{B,2} = \frac{\hat{\tau}_{A,1}}{\hat{\tau}_{B,1}}\hat{\tau}_{B,2}$$

where the node A is chosen as the reference point. Then, with different reference point, the ToF between the node A and B can be expressed in two different forms, given by

$$\begin{aligned}\hat{T}^{(A)}_{of}(A,B) &= \frac{1}{2}(\hat{\tau}_{A,2} - \frac{\hat{\tau}_{A,1}}{\hat{\tau}_{B,1}}\hat{\tau}_{B,2}) \\ &= \frac{\hat{\tau}_{A,2}\hat{\tau}_{B,1} - \hat{\tau}_{A,1}\hat{\tau}_{B,2}}{2\hat{\tau}_{B,1}}\end{aligned} \tag{6a}$$

and

$$\begin{aligned}\hat{T}^{(B)}_{of}(A,B) &= \frac{1}{2}(\frac{\hat{\tau}_{B,1}}{\hat{\tau}_{A,1}}\hat{\tau}_{A,2} - \hat{\tau}_{B,2}) \\ &= \frac{\hat{\tau}_{A,2}\hat{\tau}_{B,1} - \hat{\tau}_{A,1}\hat{\tau}_{B,2}}{2\hat{\tau}_{A,1}}\end{aligned} \tag{6b}$$

where $\hat{T}^{(X)}_{of}(A,B)$ denotes the ToF obtained by correcting the ToF estimation with the clock of X. Next, (6a) and (6b) is normalized as

$$\hat{T}_{of}(A,B) = \frac{\hat{\tau}_{A,2}\hat{\tau}_{B,1} - \hat{\tau}_{A,1}\hat{\tau}_{B,2}}{\hat{\tau}_{A,1} + \hat{\tau}_{B,1}}. \tag{7}$$

It can be verified that the error of (7) is smaller than the greater one of (6a) and (6b). Likewise, the ToF between the node A and C can be described as

$$\hat{T}_{of}(A,C) = \frac{\hat{\tau}_{C,1}(\hat{\tau}_{A,2} + \hat{\tau}_{A,3}) - \hat{\tau}_{A,1}(\hat{\tau}_{C,2} + \hat{\tau}_{C,3})}{\hat{\tau}_{A,1} + \hat{\tau}_{C,1}} \tag{8}$$

The derivation method of $\hat{T}_{of}(B,C)$ is slightly different. According to the proposed protocol in Fig. 1, we note that

$$T_{of}(A,B) + T_{of}(B,C) - T_{of}(A,C) = \tau_{B,3} - \tau_{C,3} \tag{9}$$

which can be roughly estimated by

$$\hat{T}_{of}(A,B) + \hat{T}_{of}(B,C) - \hat{T}_{of}(A,C) = \hat{\tau}_{B,3} - \hat{\tau}_{C,3}. \tag{10}$$

Similar to (6a) and (6b), by correcting (10) with the clocks of A, B and C respectively, we obtain the following estimation,

$$\begin{aligned}\hat{T}^{(A)}_{of}(B,C) = &\frac{\hat{\tau}_{A,1}}{\hat{\tau}_{B,1}}\hat{\tau}_{B,3} - \frac{\hat{\tau}_{A,1}}{\hat{\tau}_{C,1}}\hat{\tau}_{C,3} \\ &+ \hat{T}^{(A)}_{of}(A,C) - \hat{T}^{(A)}_{of}(A,B),\end{aligned} \tag{11a}$$

$$\begin{aligned}\hat{T}^{(B)}_{of}(B,C) = &\hat{\tau}_{B,3} - \frac{\hat{\tau}_{B,1}}{\hat{\tau}_{C,1}}\hat{\tau}_{C,3} \\ &+ \hat{T}^{(B)}_{of}(A,C) - \hat{T}^{(B)}_{of}(A,B),\end{aligned} \tag{11b}$$

$$\begin{aligned}\hat{T}^{(C)}_{of}(B,C) = &\frac{\hat{\tau}_{C,1}}{\hat{\tau}_{B,1}}\hat{\tau}_{B,3} - \hat{\tau}_{C,3} \\ &+ \hat{T}^{(C)}_{of}(A,C) - \hat{T}^{(C)}_{of}(A,B).\end{aligned} \tag{11c}$$

Then, with substitution and derivation, we obtain 3 fractions which have same numerators. After simplification and normalization, we have

$$\hat{T}_{of}(B,C) = \frac{\hat{\tau}_{A,1}\hat{D}_1 + \hat{\tau}_{B,1}\hat{D}_2}{2(\hat{\tau}_{A,1}\hat{\tau}_{B,1} + \hat{\tau}_{A,1}\hat{\tau}_{C,1} + \hat{\tau}_{B,1}\hat{\tau}_{C,1})}, \tag{12}$$

where $\hat{D}_1$ and $\hat{D}_2$ represent $(\hat{\tau}_{B,1}\hat{\tau}_{C,2} - \hat{\tau}_{C,1}\hat{\tau}_{B,2})$ and $(\hat{\tau}_{C,1}\hat{\tau}_{A,3} - \hat{\tau}_{A,1}\hat{\tau}_{C,3})$, respectively. Therefore, we obtain all ToFs of a three-active network.

By analogy, we can also derive calculating formulas for the ToFs of an N-active network. According to the protocol among multiple nodes shown in Fig. 2, there are

$$T_{of}(A,\beta) = \frac{1}{2}(\sum_{k\in\mathcal{L}_\beta}\tau_{A,k} - \sum_{k\in\mathcal{L}_\beta}\tau_{\beta,k}) \tag{13}$$

and

$$T_{of}(A,\alpha) + T_{of}(\alpha,\beta) - T_{of}(A,\beta) = \sum_{k\in\mathcal{L}_\beta}\tau_{\beta,k} - \sum_{k\in\mathcal{L}_\beta}\tau_{\alpha,k}. \tag{14}$$

Using the same derivation method, we obtain (3). □

Remark 1: For an N-active network, the performance of NB-TWR is same as or even better than the best performance of the best performance of the state-of-art, i.e., AltDS-TWR. We show the detailed performance analysis in Section V and numerical simulation results in Section VI.

Proposition 2: In an N-active network, for measuring all ToFs with the high precision as AltDS-TWR, $N+1$ frames are the minimum requirement.

Proof: Consider an N-active network, every active node transmits at least one frame, otherwise it degenerates to a passive node. Therefore, the number of required frames is at least equal to N. However, if each node only sends one message, it will make it impossible for any node to obtain the clock relations among the nodes. Since none of the nodes can contribute more information to the external environment after its transmitting. Then the clocks of the nodes in network cannot be synchronized. As a result, none can calculate high-precision ToF due to the large impact of clock drift. □

IV. Network Based Passive Ranging Algorithm

Based on the NB-TWR, we propose the NB-PR algorithm for the passive nodes to determine the ToF in network. It provides positioning service for passive nodes, regardless of the number of passive nodes. The specific method is illustrated as follows.

The communication protocol of NB-PR is shown in Fig. 3 and the calculation formula is shown in (15). With the timestamps obtained, the passive node χ determines the difference of the ToF $T_{of}(\beta,\chi)$ and $T_{of}(\alpha,\chi)$, i.e., $T_{of}(\beta,\chi) - T_{of}(\alpha,\chi)$. The execution steps of the algorithm can be simply divided into three steps. Firstly, we perform NB-TWR. Secondly, the node A transmits the last message, which is represented by dotted line in Fig. 3. Thridly, the passive node χ determines all $T_{of}(\beta,\chi) - T_{of}(\alpha,\chi)$ with fomula (15). Due to the space limitation, we eliminate the detailed derivation and only provide a brief derivation of the formula as below.

Proposition 3: With NB-PR, the general formula of calculating ToF difference is (15).

Proof: Firstly, according to the protocol in Fig. 3, we have

$$T_{of}(A,\beta) + T_{of}(\beta,\chi) - T_{of}(A,\chi) = \sum_{k\in\mathcal{L}_\beta}\tau_{\chi,k} - \sum_{k\in\mathcal{L}_\beta}\tau_{\beta,k}, \tag{16}$$

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$$\hat{T}_{of}(\beta,\chi)-\hat{T}_{of}(\alpha,\chi)= \begin{cases} \dfrac{3\sum\limits_{k\in\mathcal{L}_\beta}(2\hat\tau_{A,1}\hat\tau_{\beta,1}\hat\tau_{\chi,k}-\hat\tau_{A,1}\hat\tau_{\chi,1}\hat\tau_{\beta,k}-\hat\tau_{\beta,1}\hat\tau_{\chi,1}\hat\tau_{A,k})}{2(\hat\tau_{A,1}\hat\tau_{\chi,1}+\hat\tau_{\beta,1}\hat\tau_{\chi,1}+\hat\tau_{A,1}\hat\tau_{\beta,1})}, & \mathbb{N}(\beta)\geq\mathbb{N}(\alpha)=1 \\ \dfrac{2\left[\hat\tau_{A,1}\hat\tau_{\chi,1}\sum\limits_{k\in\mathcal{L}_\alpha}(\hat\tau_{\beta,1}\hat\tau_{\alpha,k}-\hat\tau_{\alpha,1}\hat\tau_{\beta,k})+\hat\tau_{\alpha,1}\sum\limits_{k\in\{\mathcal{L}_\beta/\mathcal{L}_\alpha\}}(2\hat\tau_{A,1}\hat\tau_{\beta,1}\hat\tau_{\chi,k}-\hat\tau_{A,1}\hat\tau_{\chi,1}\hat\tau_{\beta,k}-\hat\tau_{\beta,1}\hat\tau_{\chi,1}\hat\tau_{A,k})\right]}{\hat\tau_{A,1}\hat\tau_{\alpha,1}\hat\tau_{\beta,1}+\hat\tau_{A,1}\hat\tau_{\alpha,1}\hat\tau_{\chi,1}+\hat\tau_{A,1}\hat\tau_{\beta,1}\hat\tau_{\chi,1}+\hat\tau_{\alpha,1}\hat\tau_{\beta,1}\hat\tau_{\chi,1}}, & \mathbb{N}(\beta)\geq\mathbb{N}(\alpha)>1 \\ \hat{T}_{of}(\alpha,\chi)-\hat{T}_{of}(\beta,\chi), & \mathbb{N}(\beta)<\mathbb{N}(\alpha) \end{cases} \tag{15}$$

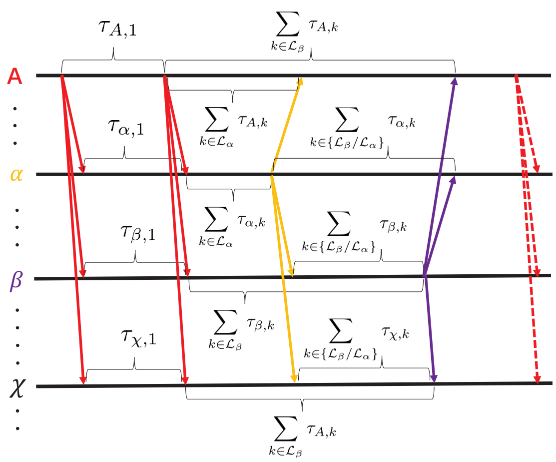

Fig. 3: The protocol of NB-PR is based on NB-TWR (A, α, β are active nodes and χ is a passive node). Compared with NB-TWR, one more message (dotted line) is transmitted from A after all active nodes completing message exchanges. This frame contains all timestamps that the node A has obtained at that time.

Similiar to the method used in Section III, we can correct the estimation with the clocks of A, β and χ, respectively. Then, we obtain three similar formulas as

$$\hat{T}_d^{(A)}(A,\chi,\beta)=\frac{\hat\tau_{A,1}}{\hat\tau_{\chi,1}}\sum_{k\in\mathcal{L}_\beta}\hat\tau_{\chi,k}-\frac{\hat\tau_{A,1}}{\hat\tau_{\beta,1}}\sum_{k\in\mathcal{L}_\beta}\hat\tau_{\beta,k}-\hat{T}_{of}^{(A)}(A,\beta), \tag{17a}$$

$$\hat{T}_d^{(\beta)}(A,\chi,\beta)=\frac{\hat\tau_{\beta,1}}{\hat\tau_{\chi,1}}\sum_{k\in\mathcal{L}_\beta}\hat\tau_{\chi,k}-\sum_{k\in\mathcal{L}_\beta}\hat\tau_{\beta,k}-\hat{T}_{of}^{(\beta)}(A,\beta), \tag{17b}$$

$$\hat{T}_d^{(\chi)}(A,\chi,\beta)=\sum_{k\in\mathcal{L}_\beta}\hat\tau_{\chi,k}-\frac{\hat\tau_{\chi,1}}{\hat\tau_{\beta,1}}\sum_{k\in\mathcal{L}_\beta}\hat\tau_{\beta,k}-\hat{T}_{of}^{(\chi)}(A,\beta), \tag{17c}$$

where $T_d(A,\chi,\beta)$ denotes the ToF difference $(T_{of}(\beta,\chi)-T_{of}(A,\chi))$. After further simplification and normalization, they can be written as the case where α represents A, i.e., case $\mathbb{N}(\beta)\geq\mathbb{N}(\alpha)=1$ in (15). As for the case where α and β are neither A, i.e., $\mathbb{N}(\beta)\geq\mathbb{N}(\alpha)>1$. According to Fig. 3, we have

$$T_{of}(\beta,\chi)+T_{of}(\alpha,\beta)-T_{of}(\alpha,\chi)=\sum_{k\in\{\mathcal{L}_\beta/\mathcal{L}_\alpha\}}\tau_{\chi,k}-\sum_{k\in\{\mathcal{L}_\beta/\mathcal{L}_\alpha\}}\tau_{\beta,k}. \tag{18}$$

Then, similiarly, we can correct the estimations with the clock of A, α, β and χ respectively. To avoid repetitive and lengthy expressions, we eliminate these four expressions here. After simplification and normalization, we can obtain the final result shown in case $\mathbb{N}(\beta)\geq\mathbb{N}(\alpha)>1$ in (15). □

Algorithm 2 Network based passive ranging

Input: The input of NB-TWR, an additional frame, the id set of passive nodes $\mathcal{P}$, preset delay T_{Last}.
Output: Estimated ToF difference $\hat{T}_{of}(\beta,\chi)-\hat{T}_{of}(\alpha,\chi)$ where $\alpha,\beta\in\mathcal{C}$, $\chi\in\mathcal{P}$.
Run NB-TWR;
Delay T_{Last};
A transmits a message;
Calculate all $\hat{T}_{of}(\beta,\chi)-\hat{T}_{of}(\alpha,\chi)$ where $\alpha,\beta\in\mathcal{C}$, $\chi\in\mathcal{P}$ by fomula (15);
return All $\hat{T}_{of}(\beta,\chi)-\hat{T}_{of}(\alpha,\chi)$;

Remark 2: We conclude that, in an N-active network, after performing NB-PR, the passive node χ can obtain $\binom{N}{2}$ hyperbolic branches in total using the acquired ToF differences. The focuses of each hyperbolic branches are at the locations of two active nodes. The principle has been proved in [8]. The location of χ is at their intersection. Then, by mathematical means we can inversely figure out all ToFs between the passive nodes and the active nodes. More generally, if an active node ignores the message it has or hasn't transmitted, it will essentially be regarded as a passive node and can work as a passive node as well. Consequently, this algorithm is an indirect ranging method without the limitation of the number of passive nodes. It should be noted that in 2-D scenarios, at least three active nodes are required to achieve the localization, while in 3-D scenarios, at least four are required.

V. Performance Analysis

A. The Error Analyses of NB-TWR and NB-PR

In typical cases, we consider clock drift as the main factor causing error. According to the system model provided in Section II. For convience, we define the errors of ToF and ToF difference as $\hat{e}(\alpha,\beta)$ and $\hat{e}(\alpha,\chi,\beta)$ respectively. Then, we can figure out the experssions of the errors in different cases in (3) and (15) as follows.

1) NB-TWR Case: $\mathbb{N}(\beta)\geq\mathbb{N}(\alpha)=1$ *(α is A):*

$$\begin{aligned}\hat{e}(A,\beta)&=\hat{T}_{of}(A,\beta)-T_{of}(A,\beta)\\ &=\Big(\frac{2}{\frac{1}{k_A}+\frac{1}{k_\beta}}-1\Big)\frac{\sum\limits_{k\in L_\beta}(\tau_{\beta,1}\tau_{A,k}-\tau_{A,1}\tau_{\beta,k})}{\tau_{A,1}+\tau_{\beta,1}}\\ &=\Big(\frac{2}{\frac{1}{k_A}+\frac{1}{k_\beta}}-1\Big)T_{of}(A,\beta).\end{aligned} \tag{19}$$

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TABLE I: Comparison of the number of frames required

Number n	NB-TWR + NB-PR	AltDS-TWR + PER
2	4	4
10	12	136
100	102	14851
1000	1002	1498501
N	$N+2$	$3\binom{N}{2}+1$

2) NB-TWR Case: $\mathbb{N}(\beta)\geq\mathbb{N}(\alpha)>1$ *(α, β are neither A) :*

$$\hat{e}(\alpha,\beta)=(\frac{3}{\frac{1}{k_A}+\frac{1}{k_\alpha}+\frac{1}{k_\beta}}-1)T_{of}(\alpha,\beta). \tag{20}$$

3) NB-PR Case: $\mathbb{N}(\beta)\geq\mathbb{N}(\alpha)=1$ *(α is A):*

$$\begin{aligned}&\hat{e}(A,\chi,\beta)\\ =&(\hat{T}_{of}(\beta,\chi)-\hat{T}_{of}(A,\chi))-(T_{of}(\beta,\chi)-T_{of}(A,\chi))\\ =&(\frac{3}{\frac{1}{k_A}+\frac{1}{k_\beta}+\frac{1}{k_\chi}}-1)T_d(\alpha,\chi,\beta).\end{aligned} \tag{21}$$

4) NB-PR Case: $\mathbb{N}(\beta)\geq\mathbb{N}(\alpha)>1$ *(α, β are neither A) :*

$$\hat{e}(\alpha,\chi,\beta)=\quad(\frac{4}{\frac{1}{k_A}+\frac{1}{k_\alpha}+\frac{1}{k_\beta}+\frac{1}{k_\chi}}-1)T_d(\alpha,\chi,\beta), \tag{22}$$

where $T_d(\alpha,\chi,\beta)$ denotes $T_{of}(\beta,\chi)-T_{of}(A,\chi)$. These error expressions are different from each other, since their derivation processes use different number of clocks to correct the estimations.

Remark 3: Quantificationally, consider a scenario where T_{of} or T_d corresponds to the distance of 5 km and the largest clock deviation is ± 20 ppm. According to the expressions of errors shown above, we can conclude that the worst-case error is 10 cm. Compared with the algorithm AltDS-TWR and PER which are provided in [8] and [13] respectively, the worst-case errors of our algorithms are all on the same order of magnitude as those of them. Furthermore, according to (20), (21) and (22), obviously our estimation results are harder to lead to the worst case. This is result from the use of the information of multiple clocks. Therefore, compared with the existing algoritms, the absolute error of our algorithms have lower mean in multiple measurements as shown in Section VI. Likewise, if we average more synchronization time on different clocks to eliminate the impact of clock drift, it will be harder to lead to the worst case. But the value of the worst-case error remains unchanged.

B. Frame Cost of NB-TWR and NB-PR

In this part, we compare the number of frames required by our algorithms with that required by existing algorithms. For an N-active network, NB-TWR requires $N+1$ frames according to Section III. If we implement NB-PR at the same time, it requires $N+1+1$ frames in total. As a contrast, we extend traditional algorithms AltDS-TWR and PER to multiple node cases by making each node in the network play the role of the anchors in turn. Since measuring a ToF requires 3 frames, it can be figured that AltDS-TWR requires $3\binom{N}{2}$ frames. With PER, it requires one more frame to assist the passive nodes. Let n denote the number of active nodes in the network. Tab. I shows that our algorithm is much more efficient than existing works and validate the necessity of our work.

VI. Numerical Simulation Results

In this section, we present numerical simulation results of the proposed algorithms. Consider a 2-D scenario where 10 active nodes and a passive node are randomly distributed in a 10 km×10 km square plane. According to the model in Part II, e_X is modeled as a variable randomly distributed in certain interval $[-e_{max},+e_{max}]$. To control the variables, we set the total measurement time to 136 ms. Then, we implement the traditional algorithms and our proposed algorithms respectively to compare their performances.

A. Performance in Typical Scenarios

Firstly, while implementing the proposed algorithms, without loss of generality, we choose a node randomly as the device A to perform the algorithms NB-TWR and NB-PR. Twelve frames are used to complete the measurement of all 45 ToFs and 45 ToF differences. Our algorithms are compared with AltDS-TWR and PER, which requires 136 frames in total to complete the same task. The performance are evaluated as

$$R=\mathcal{V}[\sum\hat{e}^2(\alpha,\beta)/(N^2-N)]^{1/2} \tag{23}$$

and

$$R_\chi=\mathcal{V}[\sum\hat{e}^2(\alpha,\chi,\beta)/(N^2-N)]^{1/2} \tag{24}$$

where N, $\mathcal{V}$ and $\sum$ denote the number of active nodes, the speed of light and $\sum_{\alpha,\beta}$, respectively. In essence, (23) is the root mean square error (RMSE) of $\binom{N}{2}$ ToFs and (24) is the RMSE of $\binom{N}{2}$ ToF differences in the N-active network. Hence, they can reflect the average performance of the network ranging.

To analyze the impact of clock drift, we set the maximum of clock deviation e_{max} to $[0:5:100]$ ppm. For every e_{max}, we have 10,000 independent repetitive simulations. The average results are presented in Fig. 4 and 5. As shown in figures, the numerical results coincidence with those of theoretical analysis. It can be seen that, in most cases, our algorithms have better performances. This proves the effectiveness of our algorithms from a numerical point of view. Besides, the error of NB-TWR and NB-PR have lower mean and it is explained in Section V.

B. Performance in Noisy Environments

In addition, it is reasonable to consider the measurement noise on timestamps. Let $n_{X,i}$ denote the measurement noise on the timestamp $\hat{t}_{X,i}$. We simply replace every $\hat{t}_{X,i}$ with $\hat{t}_{X,i}+n_{X,i}$. Then, $n_{X,i}$ is modeled as a random variable distributed in a certain interval $[-n_{max},+n_{max}]$. We perform four-group simulation experiments and set n_{max} to 10 ps, 150 ps, 300 ps and 500 ps, respectively. The results are shown in Fig. 6 and 7. It can be seen that within a specific range, with the increasing e_{max}, the impact of measurement noise on our algorithms is relatively stable. Note that the NB-PR is more sensitive to noise than NB-TWR. This is because the estimations of ToF difference use more timestamps, which leads to greater impact of noise.

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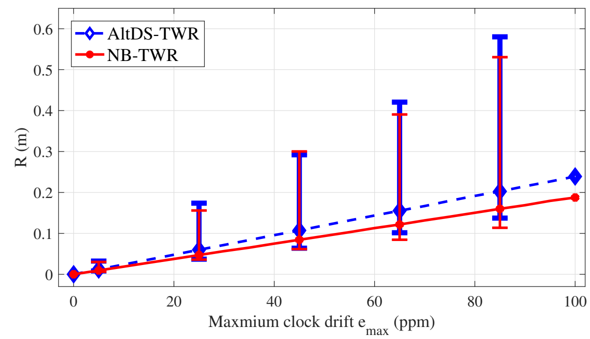

Fig. 4: Error comparison between AltDS-TWR and NB-TWR.

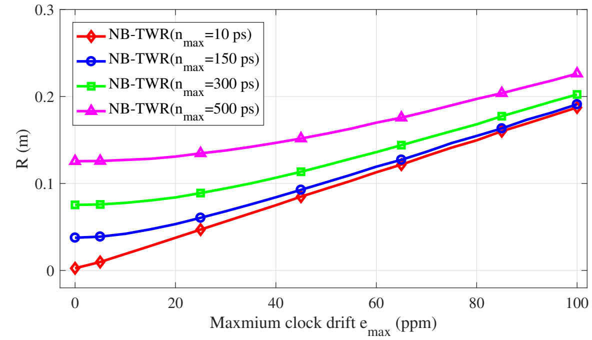

Fig. 6: RMSE of NB-TWR as a function of maximum clock drift.

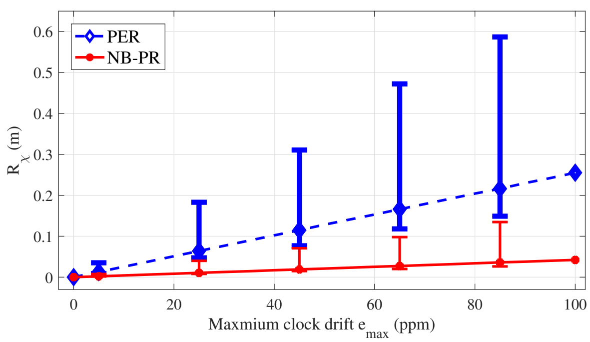

Fig. 5: Error comparison between PER and NB-PR.

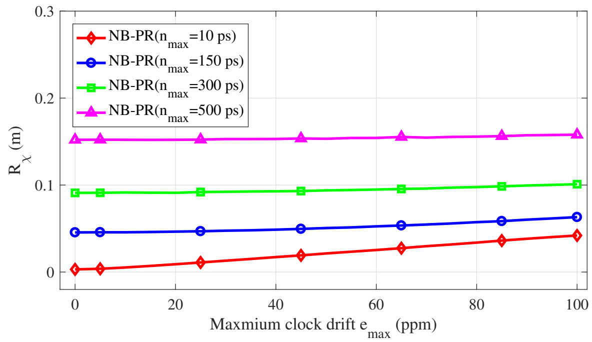

Fig. 7: RMSE of NB-PR as a function of maximum clock drift.

VII. Conclusion

This paper proposed a novel communication protocol, and further developed frame-efficient NB-TWR and NB-PR ranging algorithms. With the NB-TWR, an N-active network finishes internode ranging only with $N+1$ frames. With the NB-PR, N active nodes with unlimited number of passive nodes are capable of self-localization. Movever, through theoretical analysis, we showed that the worst-case errors of these two methods are no worse than those of the state-of-art algorithms, i.e., AltDS-TWR and PER. Numerical results validated efficiency and effectiveness of proposed algorithms. Our results can serve as guidelines for wireless networks to realize network ranging with frame-efficient, high-precision, quick update rate and simple scalability.

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